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Finite Amplitude Wave Propagation in Anisotropic Materials

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Often, ultrasound used in nondestructive evaluation is applied to materials that are elastically anisotropic. A few example materials include composites, welds, and rolled metal plates. The influence of elastic anisotropy on the propagation of ultrasound in materials that are linearly elastic is well understood. For example, elastic constants of a composite can be determined reasonably well by measuring phase velocities for propagation in certain directions. However, the influence of elastic anisotropy on nonlinear ultrasonic techniques has received much less attention.

In this work, finite amplitude bulk wave propagation is considered for materials with general elastic anisotropy of the second-, third-, and fourth-order elastic constants (anisotropy associated with triclinic symmetry). Three displacement solutions are obtained for arbitrary propagation directions of the three possible bulk wave modes (one quasi-longitudinal and two quasi-transverse). The solution corresponding to each wave mode is a harmonic series having contributions from the fundamental, second-, and third-harmonic waves. The second-harmonic wave amplitude is a function of the quadratic (β) nonlinearity parameter while the third-harmonic amplitude is a function of both the quadratic and cubic (γ) nonlinearity parameters. β is given in terms of displacement and propagation directions along with elastic tensors that define the second- and third-order elastic constants of the material. An additional contribution from the elastic tensor defining the fourth-order elastic constants is needed to define γ . Closed-form evaluation of β and γ for the three different wave modes has been conducted for a variety of materials having different crystallographic point group symmetries. Surfaces will be presented for selected materials, which illustrate the three-dimensional spatial distribution of β and γ for any propagation direction of the fundamental wave. The vanishing of β for shear waves propagating within material planes of symmetry causes the surface to display the symmetry of the material elegantly. Lastly, straightforward expressions for β and γ are given for some pure mode directions.

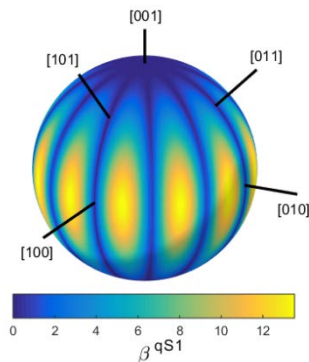


Figure 1. Distribution of the quadratic nonlinearity parameter β as a function of the propagation direction for a shear wave in a single crystal of titanium.